**Alq J Sci.** 2025;1(2):207-216
https://doi.org/10.69667/ajs.25209

*Alqalam Journal of Science*
مجلـــة القلم للعلـوم

https://alqalam.utripoli.edu.ly/index.php/AR

# Well Test Interpretation with TDS Technique: Detection and Absence of Infinite Acting Radial Flow, Case Study

**Laila Saleh*, Adel Traki, Hayat Alhaj, Marwa Gergab, Malak Al-Osta**

*Department of Petroleum Engineering, Faculty of Engineering, the University of Tripoli, Libya*
***Corresponding email**: ladasa08@gmail.com*

**Abstract**
Tiab's Direct Synthesis (TDS) technique is one of the transit pressure analysis techniques. It provides a direct approach to interpret transit well pressure tests without a type curve. The technique uses pressure derivatives to analyze data and compute reservoir parameters. One of the keys to analysis is identifying whether infinite acting radial flow (IARF) is observed. When IARF is clearly observed, the TDS and conventional methods allow estimation of reservoir parameters. But in real cases, IARF may not be observed or very short due to the short time of test or the influence of the outer boundary, which makes the conventional methods face limitations and the analysis becomes difficult or sometimes impossible. Thus, the present study applies the TDS technique on the Libyan oil field in both observed and unobserved IARF. Two wells are presented to illustrate the TDS technique. Based on the results, the TDS technique was a very useful and direct technique for analyzing transient pressure tests, especially when infinite acting radial flow is absent, as demonstrated in the analysis of wells.
**Keywords:** TDS technique, Pressure, Derivative, radial flow, wellbore storage

## Introduction

A test is simply time, a period during which the production of the well is measured either at the well test Equipment or in a production facility. Most well tests consist of changing the rate and observing the change in pressure caused by this change in rate. To perform a well test successfully, one must be able to measure the time, the rate, and the pressure, and control the rate of well tests. The interpretation of pressure data recorded during a well test has been used for many years to evaluate reservoir characteristics. Many methods have been proposed for the interpretation of transient tests, such as Horner's and type curves [1].

The TDS technique is a direct method to interpret transient well pressure tests without type curve matching [2,3]. This method uses log-log plots of the pressure and pressure derivative versus time. The technique is particularly useful when the early-time unit-slope line and/or the late-time infinite acting radial flow line are not well developed due to the lack of points or other reasons [2,3]. One of the problems in pressure derivative on log-log plot is that the radial flow region is not observed, TDS solved this issue by analyzing well bore storage region and read direct specific values from the plot substituting in Equations to determinate parameters that cannot be estimated from radial flow such like permeability, and skin this advantage is called (Short test) [2,3,4]. The objective of this paper is to analyze pressure transient tests applied in two Libyan oil wells using the advantages of the TDS technique.

## Methodology

Data was collected from two wells in the Libyan fields. The data was screened and analyzed by the TDS technique using Excel software. It contains two wells (X1 and X2); each well has two seniors (A and B). Analysis of the well when infinite acting radial flow is observed (long test). Analysis of the well when infinite acting radial flow isn't observed (short test). TDS Technique uses the pressure derivative function to determine the average reservoir pressure for cases, vertical well in a closed homogeneous reservoir, hydraulically fractured vertical well, and horizontal well in a closed anisotropic reservoir [4,5].

### *Application of TDS for Vertical Wells*

The TDS method is used to evaluate reservoir formation characterization, especially average reservoir pressure. This section describes the method to estimate reservoir pressure for vertical wells using the TDS technique.


***Received**: 16-07-2025 - **Accepted**: 15-09-2025 - **Published**: 21-09-2025*

**Alq J Sci.** 2025;1(2):207-216
https://doi.org/10.69667/ajs.25209
*Alqalam Journal of Science*
مجلة القلم للعلوم
https://alqalam.utripoli.edu.ly/index.php/AR

*Average Pressure from the TDS Technique*

A material balance for a slightly compressible fluid in bounded reservoirs leads to [5]:

$$\overline{PD}\ (tDA) = 2\pi tDA \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For closed circular reservoirs, it can be shown that during pseudo-steady state flow:

$$\overline{PD}(tDA) = PD(tDA) + \ln(reD) - (3/4) \ldots\ldots\ldots\ldots (2)$$

For long producing time, the pressure derivative function yields a straight line of unit slope. This line, which corresponds to the pseudo-steady state flow regime, starts at a tDA value of approximately 0 2. Differentiating and multiplying both sides of Equation 2 by tDA, the Equation of this straight line is obtained as:

$$tDA * PD` = 2\pi tDA \ldots\ldots\ldots\ldots\ldots (3)$$

Taking the ratio of Equation 2 to 3,gives:

$$\frac{\Delta PD}{tDA*PD`} = 1+\frac{1}{P}\left(\ln(reD) - \frac{3}{4}\right) \ldots\ldots\ldots\ldots (4)$$

Substituting for the dimensionless terms and solving for the average reservoir pressure, $\overline{P}$, we have the following Equation:

$$\frac{\Delta P}{t*\Delta P`} = 1 + \frac{141.2\ qB\mu}{kh(Pi-\overline{P})}\left(\ln(reD) - \frac{3}{4}\right) \ldots\ldots\ldots\ldots (5)$$

The average reservoir pressure for a single well located in the center of a circular reservoir using the log-log plot of pressure and pressure derivative is obtained by solving Equation 6 for $\overline{P}$.

$$p = Pi - \frac{141.2\ qB\mu}{kh}\left[\frac{(t*P`)pss}{(\Delta P)pss*(t*P`)pss}\right]\left[\ln(reD) - \frac{3}{4}\right] \ldots\ldots (6)$$

Where: Pi is the initial pressure. In some cases, we can approximate Pi to P * (ΔP)pss and (t*ΔP)pss are values of (ΔP) and (t*ΔP`) on the pseudo-steady state straight line, respectively, which are shown in (Figure 1).

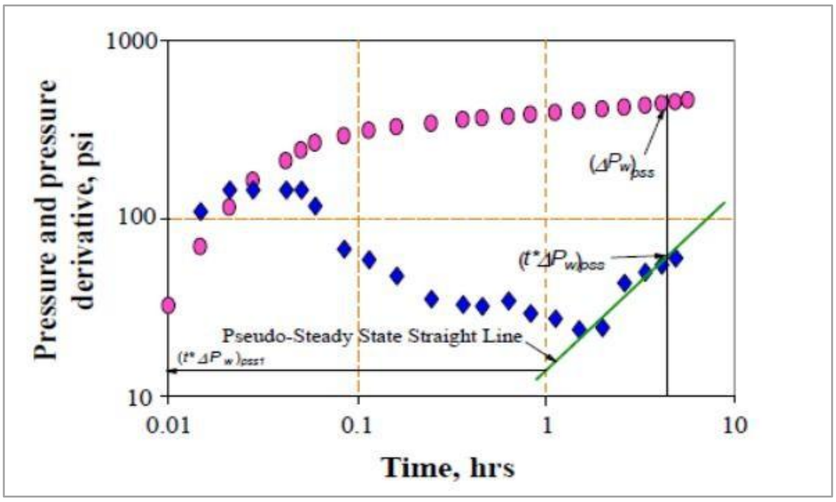


*Figure 1: the schema for Pss straight line extrapolate to t= 1hr [4]*

*TDS Procedures in Vertical Wells (work steps)*

Plot ΔP and t×ΔP` versus time on a log-log graph. Draw the infinite-acting radial flow line using late-time pressure derivative points. This line is, of course, horizontal. Read the value (t×ΔP`)r. Select any convenient time tr during the infinite-acting radial flow line and read ΔPr from the pressure curve. Draw the unit-slope line corresponding to the wellbore storage flow regime using early-time pressure and pressure derivative points. (Figure 2) shows the previous steps.


**Received**: 16-07-2025 - **Accepted**: 15-09-2025 - **Published**: 21-09-2025

**Alq J Sci.** 2025;1(2):207-216
https://doi.org/10.69667/ajs.25209
*Alqalam Journal of Science*
مجلة القلم للعلوم
https://alqalam.utripoli.edu.ly/index.php/AR

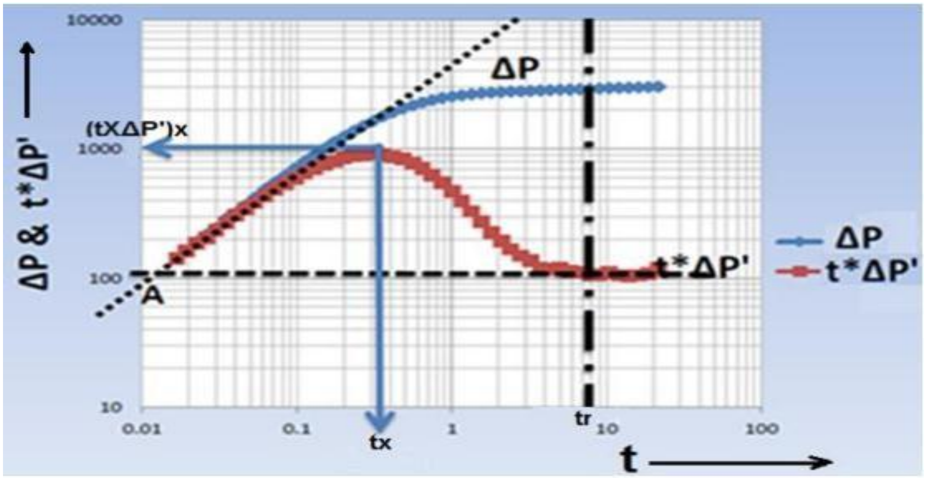


*Figure 2.: ΔP and ΔP`*t versus time (log-log plot) [4]*

1) Calculate the wellbore storage coefficient by the following Equation:

$$C = \frac{qB}{24} * \frac{tx}{\Delta Px} \quad \text{.............................................. (7)}$$

2) Calculate permeability using the following Equation:

$$K = \frac{70.6q\mu B}{h*(t*\Delta p`)r} \quad \text{.................................................... (8)}$$

3) Calculate skin and pressure drop due to skin by the following Equations:

$$S = 0.5\left[\frac{\Delta Pr}{(t*\Delta P)r} - \ln\left(\frac{k*tr}{\emptyset\mu Ctrw^2}\right) + 7.43\right] \quad \text{.................................. (9)}$$

$$\Delta Ps = 2 * (t * \Delta P)r * S \quad \text{............................................ (10)}$$

*Faults and Bounded Reservoirs Calculations*

In the case of a sealing fault, the pressure behavior is influenced by the limit of fluid flow. It is necessary to calculate flow efficiency and the distance between the well and the fault as follows;

4) Calculate the distance to a fault by the Equation: -

$$D = 0.0122\left(\frac{Kt_x}{\phi\mu c_t}\right)^{1/2} \quad \text{..........................(11)}$$

5) Calculate flow efficiency by the Equation: -

$$FE = \frac{\bar{P} - pwf - \Delta ps}{\bar{P} - pwf} \quad \text{.....................................(12)}$$

## Results and Discussion

*Case 1: Analysis of pressure data for well X1*

Well X1, the well was tested in 2004. At the beginning of the test, the rate was adjusted, then the well was flowed at different choke sizes, after which the well was shut-in for the final build-up. An analysis was performed on the pressure buildup data collected from the above-mentioned well X1. The goal of the test is to determine the permeability (K), skin(S), and average reservoir pressure ($\bar{P}$). The well was producing at a rate of about 917 STB/day oil. A producing time of 72.320 hours was calculated since the well was first produced.

*Case A- Infinite acting radial flow line is observed (Long test) for Well X1*

Step (1): Prepare General Data Required for the Analysis as shown in (Tables 1 and 2).


**Received**: 16-07-2025 - **Accepted**: 15-09-2025 - **Published**: 21-09-2025

**Alq J Sci.** 2025;1(2):207-216
https://doi.org/10.69667/ajs.25209
*Alqalam Journal of Science*
مجلة القلم للعلوم
https://alqalam.utripoli.edu.ly/index.php/AR

*Table 1: Formation Data for well X1*

| | |
|---|---|
| Formation thickness, h | 129.5 ft |
| Formation porosity, ϕ | 24.29% |
| Total system compressibility, Ct | $1.59783*10^{-5}$ |

*Table 2: Well and fluid data for well X1*

| | |
|---|---|
| Oil Production rate, qo | 917 STB/D |
| Wellbore radius, rw | 0.354 ft |
| Oil formation volume factor, Bo | 1.556 bbl/STB |
| Oil viscosity, μo | 0.38 cp |
| Drainage radius, re | 1,640 ft |

Step (2): Construct a log-log plot of (Δp) and (Δp') vs. time (Figure 3).

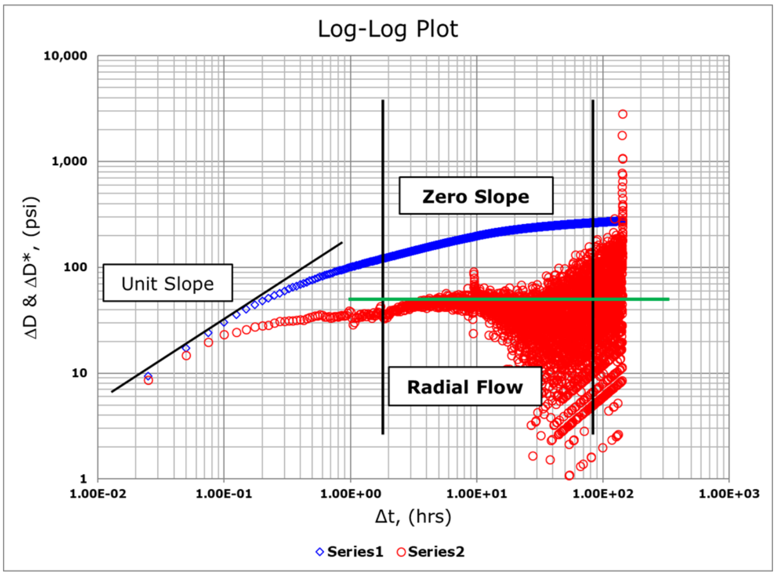


***Figure 3: Pressure and Pressure Derivative Vs Time.***

Step (3): Draw a horizontal line through the radial flow horizontal line, read the value of the pressure derivative, (t*ΔP') r, and estimate permeability by using Equation 8.

$$k = \frac{70.6 * 917 * 380 * 1.556}{129.5 * 1445.3} = 8.443 \text{ md}$$

Step (4): Read the pressure value, ΔPr, at any convenient time, tr1, during the first radial flow regime or tr2, during the second radial flow regime, and compute the skin factor by using Equation 9.

$$s = 0.5\left[\frac{200}{35} - ln\left(\frac{8.4434076 * 10}{0.2429 * 0.38 * 1.60E - 05 * 0.354^2}\right) + 7.43\right] = -3.40\ (Damage)$$

Calculation of the pressure drop due to skin effect by using Equation 10
ΔPs = 2*35*-3.40 = -238 psi


***Received**: 16-07-2025 - **Accepted**: 15-09-2025 - **Published**: 21-09-2025*

**Alq J Sci.** 2025;1(2):207-216
https://doi.org/10.69667/ajs.25209
Alqalam Journal of Science
مجلـــة القلم للعلـوم
https://alqalam.utripoli.edu.ly/index.php/AR

Step (5): Calculate the WBS coefficient using t = 0.05 hr and ΔP = 29 psi (obtained from unit-slope line) and using Equation 7.

$$C = \left(\frac{917 * 1.556}{24}\right)\frac{0.05}{29}$$

C = 0.10250 bbl/ psi

Step (6): Take any point convenient on the late pseudo steady-state flow regime and read the time, pressure, and pressure derivative: tpss, ΔPpss, and (t*ΔP) pss, and calculate the average reservoir Pressure using the following Equation 6:

$$\overline{P} = 2079.771 - \frac{141.2 * 917 * 0.38 * 1.556}{129.5 * 8.4434076}\left[\frac{35}{200-35}\right] * \left[ln\frac{1640}{0.354} - \frac{3}{4} + -3.40\right]$$

Average reservoir pressure = 1954.417 psi

Step (7): Flow Efficiency Calculation by using Equation 14.

$$FE = \left(1 - \frac{-238}{2289}\right) = 1.10$$

Step (8): Productivity Index Calculations by using the following Equation:

$$PI = \frac{7.08 * 10^{-3} * 8.4434076 * 129.5}{0.38 * 1.556 * \left(ln\left(\frac{1640}{0.354}\right) - \frac{3}{4} + (-3.40)\right)}$$

*PI* = 3.05

***Case B. Infinite acting radial flow line is not observed (Short test) for Well (X1)***

Step (1): Construct a log-log plot of pressure and pressure derivative vs. time, without the late-time points as shown in (Figure 4).

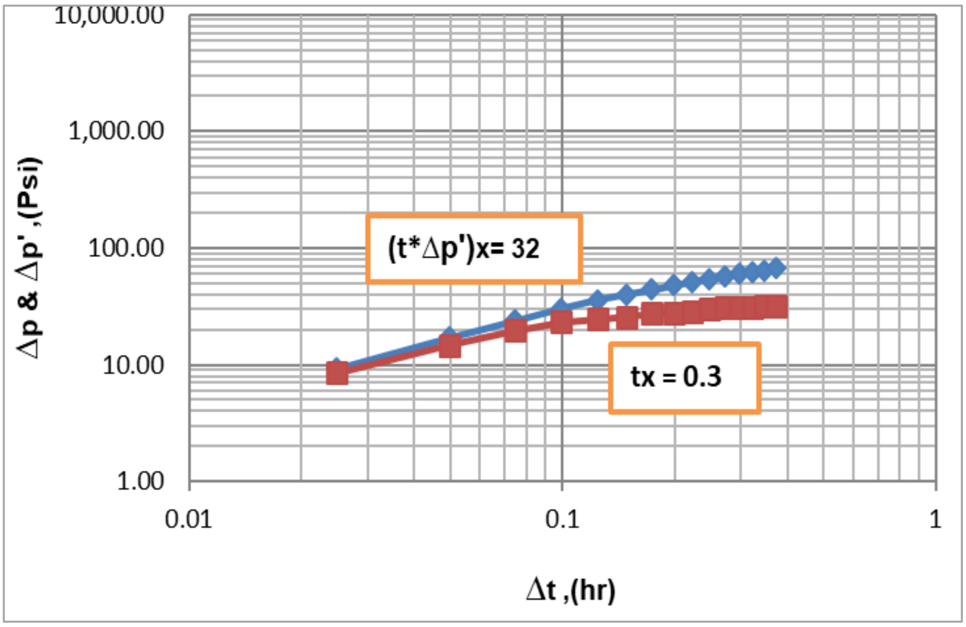


*Figure 4. Pressure and Pressure Derivative Vs Time.*

Step (2) - The unit slope line on the pressure curve is well defined.

Step (3) - The coordinates of the maximum point corresponding to the wellbore storage hump (assuming no phase segregation) are: tx = 0.3 hrs and (t×ΔP′)x = 32 psi

Step (4)- Calculate the wellbore storage coefficient using t = 0.05 hr and ΔP = 29 psi (obtained from unit-slope line) and Equation 8.


***Received**: 16-07-2025 - **Accepted**: 15-09-2025 - **Published**: 21-09-2025*

**Alq J Sci.** 2025;1(2):207-216
https://doi.org/10.69667/ajs.25209
*Alqalam Journal of Science*
مجلـــة القلم للعلـوم
https://alqalam.utripoli.edu.ly/index.php/AR

$$C = \left(\frac{917 * 1.556}{24}\right) * \frac{0.05}{29}$$

C=0.10250 bbl/ psi

Step (5) - The permeability of the formation is obtained from the coordinates of the maximum point: tx = 0.3 hr and (t×ΔP′)x =32 psi

$$(t * \Delta P`)r = \left(\frac{917 * 1.556}{70.6 * 0.10250}\right) * 0.3 - 0.75(32)$$

$$(t * \Delta P`)r = 35.1504$$

$$K = \frac{70.6 * 917 * 0.38 * 1.556}{129.5 * 35.1504}$$

K=8.41 md

Step (6) - Calculate the skin factor using equation 9.

$$S = 0.921\left(\frac{32}{35.1504}\right)^{1.1} - 0.5\ln\left(\frac{0.8938 * 0.10250}{0.2429 * 129.5 * 1.60 * 10^{-5} * 0.354^{2}}\right)$$

S= -2.80993

Calculation pressure drop due to skin effect by using the following equation.

ΔPs = 2*35.1504*-2.80993= -197.540 psi

Step (7) – Starting time of the pseudo-radial flow line the pressure test been run long enough to observe the infinite-acting line, this line would have approximately started at:

$$t_{st} = \frac{0.38 * 0.10250}{3.9 * 10^{-5} * 8.407342 * 129.5}\left[ln\left(\frac{0.8935 * 0.10250}{0.2429 * 1.60 * 10^{-5} * 129.5 * 0.354^{2}}\right)\right]$$

$$t_{st} = 6.6\ hr$$

The summary of the analysis of well X1 is shown in table 3 for both cases.

***Table 3.: Summary of results for (well X1)***

| Parameters | TDS technique | |
|---|---|---|
| | IARF (observed-long test) | IARF (Not Observed-Short Test) |
| K, md | 8.4434076 | 8.407342 |
| S | -3.40 | -2.80993 |
| ΔPs, psi | -238 | -197.540 |
| C, bbl/psi | 0.10250 | 0.10250 |
| $P$,psi | 1954.417 | - |
| FE | 1.10 | - |
| $PI$ | 3.05 | - |

***Case 2: Analysis of pressure data for well X2***

Well X2 the well was tested on October 18, 2009. At the beginning of the test, the rate was adjusted, then the well was flowed at different choke sizes, after which the well was shut-in for the final build-up. An analysis was performed on the pressure buildup data collected from the above-mentioned well X2. The goal of the test is to determine the permeability(K), skin(S), and average reservoir pressure ($\overline{P}$). The well was producing at a rate of about 1248 STB/day oil. A producing time of 100 hours was calculated since the well was first produced.

**Case A. Infinite acting radial flow line is observed (Long test) for the Well X2**

Step (1): Prepare General Data Required for the Analysis as shown in (Tables 4 and 5).


***Received**: 16-07-2025 - **Accepted**: 15-09-2025 - **Published**: 21-09-2025*

**Alq J Sci.** 2025;1(2):207-216
https://doi.org/10.69667/ajs.25209
*Alqalam Journal of Science*
مجلـــة القلم للعلــوم
https://alqalam.utripoli.edu.ly/index.php/AR

*Table 4: Formation Data for (well X2)*

| | |
|---|---|
| Formation thickness, h | 92 ft |
| Formation porosity, ϕ | 14.2 % |
| Total system compressibility, Ct | 3E-05 psi-1 |

*Table 5 : well and fluid data for (well X2)*

| | |
|---|---|
| Oil Production rate, qo | 1248 STB/D |
| Wellbore radius, rw | 0.245ft |
| Oil formation volume factor, Bo | 1.65 bbl/STB |
| Oil viscosity, μo | 0.35 cp |

Step (2): Construct a log-log plot of pressure and pressure derivative vs. time (Figure 5).

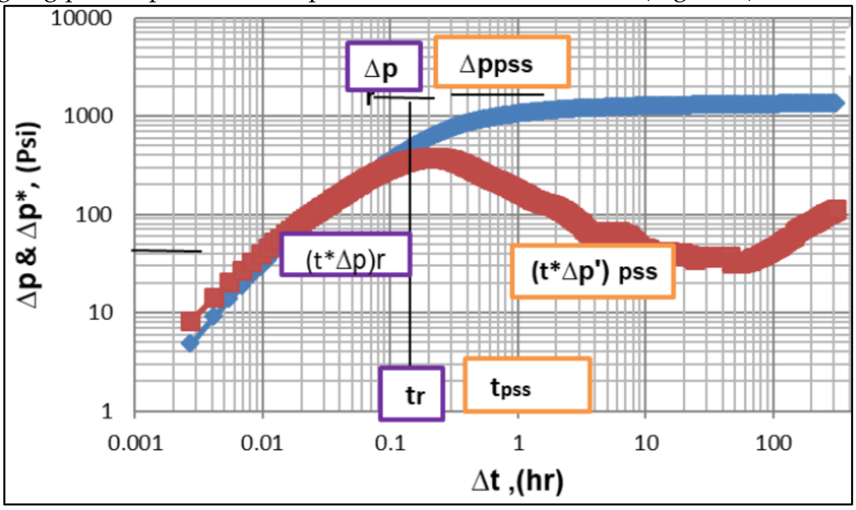


*Figure 5: Pressure and Pressure Derivative Vs Time.*

Step (3): Draw a horizontal line through the radial flow horizontal line, read the value of the pressure derivative, (t*ΔP') r, and estimate permeability by using equation 8.

$$k = \frac{70.6 * 1248 * 0.35 * 1.65}{92 * 52.6554} = 10.50366 \text{ md}$$

Step (4): Read the pressure value, ΔPr, at any convenient time, tr1, during the first radial flow regime, or tr2, during the second radial flow regime, and compute the skin factor by using Equation 9.

$$s = 0.5\left[\frac{1400}{52.6554} - ln\left(\frac{10.50366 * 8.58883}{0.142 * 0.35 * 3 * 10^{-5} * 0.245^2}\right) + 7.43\right] = 6.643361$$

Calculation of the pressure drop due to skin effect by using equation 10.
$\Delta Ps = 2 * (t * \Delta P)r * S$
ΔPs = 2*52.6554*-6.643361 = -699.6177 psi

Step (5): Calculate the WBS coefficient using t = 0.01 hr and ΔP = 40 psi (obtained from the unit-slope line and equation 7.

$$C = \left(\frac{1248 * 1.65}{24}\right)\frac{0.01}{40}$$


**Received**: 16-07-2025 - **Accepted**: 15-09-2025 - **Published**: 21-09-2025

**Alq J Sci.** 2025;1(2):207-216
https://doi.org/10.69667/ajs.25209
*Alqalam Journal of Science*
مجلة القلم للعلوم
https://alqalam.utripoli.edu.ly/index.php/AR

C=0.02145 bbl/psi

Step (6): Take any point convenient on the late pseudo steady-state flow regime and read the time, pressure, and, pressure derivative: tpss, ΔPpss, and (t*ΔP) pss, and calculate the average reservoir Pressure using Equation 6.

$$\overline{P} = 1363 - \frac{141.2*1248*0.35*1.65}{92*10.50366}\left[\frac{43.56}{1343.56-43.56}\right]*\left[ln\frac{1920}{0.245}-\frac{3}{4}+6.643361\right]$$

Average reservoir pressure = 1310.563 psi

Step (7): Flow Efficiency Calculation by using Equation 11.

$$FE = \left(\frac{1310.563 - 2029.64 - 699.6177}{1310.563 - 2029.64}\right)$$

FE= 1.9729

Step (8): determine the drainage area of the well,

$$A = \frac{1248 * 1.65}{4.27 * 0.142 * 3 * 10^{-5} * 92}\left(\frac{185.041}{43.56}\right)$$

A= 522,701,0 ft²=119.9956 acres

Step (9): The Distance to a fault using Equation 12,

$$d = 0.0122\left(\frac{10.50366 * 3.2}{0.142 * 0.35 * 3 * 10^{-5}}\right)^{0.5}$$

d =57.9 ft

***Case B. Infinite acting radial flow line is not observed (Short test) for Well (X2)***

Step (1): Construct a log-log plot of pressure and pressure derivative vs. time, without the late-time points see (Figure 6).

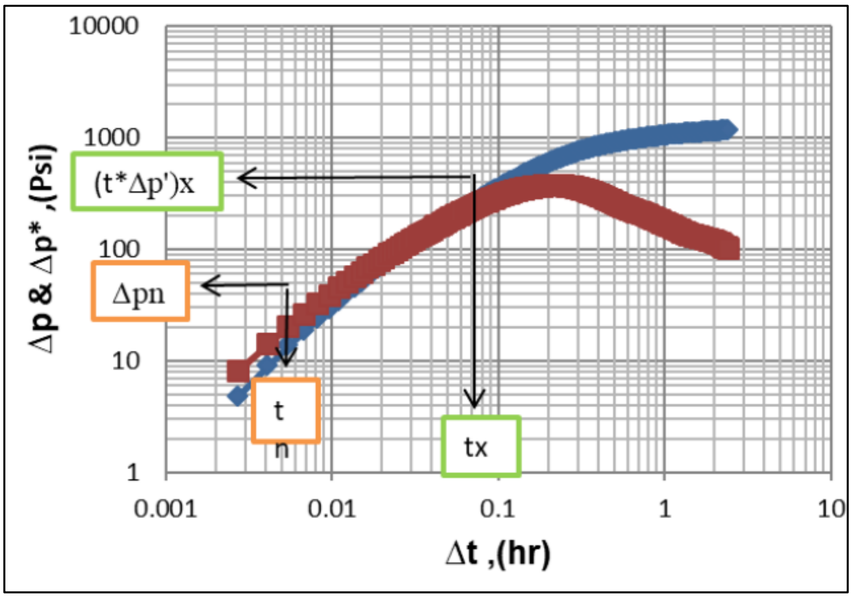


*Figure 6: Pressure and Pressure Derivative Vs Time.*

Step (2)- The unit slope line on the pressure curve is well defined.

Step (3) - The coordinates of the maximum point corresponding to the wellbore storage hump (assuming no phase segregation) are: tx = 0.26 hrs and (t×ΔP')x = 400 psi

Step (4)- Calculate the wellbore storage coefficient using t = 0.01 hr and ΔP = 40 psi (obtained from unit-slope line) and the Equation 7.

$$C = \left(\frac{1248 * 1.65}{24}\right) * \frac{0.01}{40}$$


**Received**: 16-07-2025 - **Accepted**: 15-09-2025 - **Published**: 21-09-2025

**Alq J Sci.** 2025;1(2):207-216
https://doi.org/10.69667/ajs.25209
*Alqalam Journal of Science*
مجلــة القلم للعلـوم
*https://alqalam.utripoli.edu.ly/index.php/AR*

C= 0.02145 bbl/ psi

Step (5) - The permeability of the formation is obtained from the coordinates of the maximum point: tx = 0.26 hr and (iat×ΔP')x =400 psi using the Equation 10.

$$(t * \Delta P')r = \left(\frac{1248 * 1.65}{70.6 * 0.02145}\right) * 0.26 - 0.75(400)$$

$$(t * \Delta P')r = 53.54108$$

$$K = \frac{70.6 * 1248 * 0.35 * 1.65}{92 * 53.54108}$$

K= 10.329 md

Step 6 - Calculate the skin factor using the Equation 9.

$$S = 0.921\left(\frac{400}{53.54108}\right)^{1.1} - 0.5 \ln\left(\frac{0.8938 * 0.02145}{0.142 * 92 * 3 * 10^{-5} * 0.245^2}\right)$$

S=6.607

Calculation of pressure drop due to skin effect:

ΔPs = 2*53.451*6.607= 699.24 psi

For other parameters such as average reservoir pressure, flow efficiency, and distance to fault, it is necessary to use the existing IARF (long test); in cases where IARF is not observed, only wellbore storage, permeability, and skin can be calculated.

The summary of the analysis of X2 is shown in (Table 6) for both cases.

*Table 6: Summary and comparison of results for (well 2)*

| Parameters | TDS technique | |
|---|---|---|
| | IARF (observed-long test) | IARF (Not Observed-Short Test) |
| K, md | 10.50366 | 10.3299 |
| S | 6.643361 | 6.607 |
| ΔPs, psi | 699.6177 | 699.24 |
| C, bbl/psi | 0.02145 | 0.02145 |
| Average *P*, psi | 1310.563 | - |
| FE | 1.9279 | - |
| D, ft | 57.9 | - |

## Conclusions

The TDS is an effective and direct approach to determine permeability, skin, average reservoir pressure, and other reservoir properties. The average reservoir pressure was 1954.417 psi for well X1 and 1310 psi for well X2. The TDS method showed accurate results. The values of permeability of well X1 by using (Long test) were 8.443 md and 8.40 md for the short test, and the value of total skin was -3.40 for the long test and -2.80 for the short test. The values of permeability of well X2 were 10.3299 md and 10.504 md for short and long tests, respectively. The values of total skin factor were 6.607 and 6.643 for the short and long test, respectively. Based on the comparison of the results in both cases for each well, the results of the analysis were similar. By applying TDS, we were able to analyze short data (absence of IARF) and determine important parameters (permeability, skin, and wellbore storage) that cannot be evaluated using traditional methods because they can be evaluated only when IARF is present. The TDS technique is mainly useful when the infinite acting radial flow line has not been observed or is not well defined due to a variety of reasons, such as a lack of points or a severe noise problem.


***Received****: 16-07-2025 -* ***Accepted****: 15-09-2025 -* ***Published****: 21-09-2025*

**Alq J Sci.** 2025;1(2):207-216
https://doi.org/10.69667/ajs.25209
*Alqalam Journal of Science*
مجلــة القلم للعلـوم
https://alqalam.utripoli.edu.ly/index.php/AR

## References

1. Lee JB, Rollins J, Spivey JP. Pressure transient testing. Taxes; 2003.
2. Tiab D. Analysis of pressure and pressure derivative without type-curve matching: skin and wellbore storage. J Pet Sci Eng. 1995.
3. Tiab D. Analysis of pressure and pressure derivative without matching: vertically fractured wells in closed systems. J Pet Sci Eng. 1994.
4. Chacón A, Djebrouni A, Tiab D. Determining the average reservoir pressure from vertical and horizontal well test analysis using the Tiab's direct synthesis technique. In: SPE Asia Pacific Oil & Gas Conference and Exhibition; 2004 Oct; Perth, Australia. SPE paper 88619.
5. Engler T, Tiab D. Analysis of pressure and pressure derivative without type-curve matching: horizontal well tests in anisotropic media. J Pet Sci Eng. 1996.


***Received****: 16-07-2025 -* ***Accepted****: 15-09-2025 -* ***Published****: 21-09-2025*