

Original article

A Comparative Study of Exact Solutions for the Two Nonlinear Partial Differential Equations Using the (G'/G) -Expansion Method

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Abstract

In the present work, the (G'/G) -expansion technique is applied to construct closed-form traveling-wave solutions for two nonlinear systems: the modified dispersive water-wave (MDWW) system and the variant Boussinesq system. Compared with more elaborate symbolic approaches, this technique reaches explicit, verifiable solution families with comparatively little algebraic overhead, which in turn sheds light on how these nonlinear systems behave. Because it converts an otherwise unwieldy governing equation into a tractable algebraic system, the method is well suited to producing accurate closed-form expressions. The results reported here support the view that the (G'/G) -expansion approach is a useful tool both for the theoretical study of nonlinear wave phenomena and for problems arising in applied nonlinear dynamics.

Keywords: (G'/G) -Expansion Method; Comparative Study; Modified Dispersive Water Wave (MDWW) System; Variant Boussinesq Equation.

Introduction

Among the analytical tools developed for governing equations of complicated physical systems, the (G'/G) -expansion method stands out for its systematic character. The unknown solution is expressed through a function whose logarithmic derivative satisfies a simple linear auxiliary ODE, which turns a difficult nonlinear problem into a solvable algebraic one. Owing to this transparent structure, the technique has been used across a wide range of nonlinear model equations [1–13].

Outline of the (G'/G) -expansion method for NPDEs:

Consider a governing nonlinear equation written generally as $P(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \dots) = 0$ (2.1)

in which $u = u(x, t)$ is unknown and P collects u together with its partial derivatives, both linear and nonlinear, up to the relevant order. The recipe of [14] for solving (2.1) can be broken down into six short stages, sketched below.

Step 1. Pass to a single traveling-wave coordinate,

$$u(x, t) = u(\xi), \quad \xi = \xi(x, t) \quad (2.2)$$

which turns (2.1) into an ordinary differential equation in $u(\xi)$ alone:

$$P(u, u', u'', \dots) = 0 \quad (2.3)$$

where the prime denotes $\frac{d}{d\xi}$.

Step 2. Posit a truncated series expansion for the unknown in powers of the ratio G'/G ,

$$u(\xi) = \sum_{i=0}^n a_i \left(\frac{G'}{G}\right)^i \quad (2.4)$$

in which the auxiliary function $G = G(\xi)$ is required to satisfy the linear second-order equation

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \quad (2.5)$$

and the constants a_i ($i = 0, 1, \dots, n$), together with λ and μ , remain undetermined at this stage.

Step 3. Determine the truncation order n by comparing degrees: the degree contributed by the highest derivative must equal that of the strongest nonlinear term in (2.3). Symbolically,

$$\text{If } D[u(\xi)] = n, \Rightarrow D\left[u^r \left(\frac{d^q u}{d\xi^q}\right)^s\right] = nr + s(q + n),$$

D stands for the degree operator.

Step 4. Insert (2.4) into (2.3), invoke (2.5) to eliminate higher derivatives of G , and collect the resulting expression by powers of G'/G . Setting each power's coefficient to zero yields a polynomial system in the unknowns a_i, λ, μ, c and k .

Step 5. This polynomial system is then solved by hand for small cases or with symbolic software such as Maple or Mathematica, otherwise recovering explicit values for a_i, λ, μ, c and k .

Step 6. Substituting these values, along with the known closed-form solution of (2.5), back into the ansatz (2.4) produces the sought traveling-wave solution of the original equation (2.1).

3. Application of the method:

The MDWW system [15]:

As a first application, consider the pair of coupled equations known as the modified dispersive water-wave (MDWW) system:

$$u_t = -\frac{1}{4}v_{xx} + \frac{1}{2}(uv)_x, \quad (3.1.1)$$

$$v_t = -u_{xx} - 2uu_x + \frac{3}{2}vv_x. \quad (3.1.2)$$

Introducing the traveling-wave variables

$$u(x, t) = u(\xi), v(x, t) = v(\xi), \xi = k(x + \omega t), \quad (3.1.3)$$

converts (3.1.1)–(3.1.2) into a coupled ODE system:

$$-\frac{1}{4}kv'' + \frac{1}{2}(uv)' - \omega u' = 0, \quad (3.1.4)$$

$$-ku'' - 2uu' + \frac{3}{2}vv' - \omega v' = 0, \quad (3.1.5)$$

Here k denotes the wavenumber and ω the wave speed. Integrating (3.1.4)–(3.1.5) once with respect to ξ gives

$$k_1 - \frac{1}{4}kv' + \frac{1}{2}(uv) - \omega u = 0, \quad (3.1.6)$$

$$k_2 - ku' - u^2 + \frac{3}{4}v^2 - \omega v = 0, \quad (3.1.7)$$

with k_1, k_2 arising as integration constants. We seek solutions of (3.1.6)–(3.1.7) in the polynomial form

$$u(\xi) = \sum_{i=0}^n a_i \left(\frac{G'}{G}\right)^i, \quad (3.1.8)$$

$$v(\xi) = \sum_{i=0}^m b_i \left(\frac{G'}{G}\right)^i \quad (3.1.9)$$

with $G(\xi)$ again governed by (2.5). Comparing degrees between the derivative and nonlinear contributions in (3.1.6)–(3.1.7) pins down $n = m = 1$, leaving

$$u(\xi) = a_0 + a_1 \left(\frac{G'}{G}\right), a_1 \neq 0, \quad (3.1.10)$$

$$v(\xi) = b_0 + b_1 \left(\frac{G'}{G}\right), b_1 \neq 0, \quad (3.1.11)$$

Plugging (3.1.10)–(3.1.11), together with (2.5), into (3.1.6)–(3.1.7) and equating coefficients of like powers of (G'/G) to zero gives the algebraic system:

$$\begin{aligned} \left(\frac{G'}{G}\right)^0: & \omega a_0 + \frac{a_0 b_0}{2} + \frac{1}{4}k\mu b_1 + k_1 = 0, \\ \left(\frac{G'}{G}\right)^1: & \omega a_1 + \frac{a_1 b_0}{2} + \frac{1}{4}k\lambda b_1 + \frac{a_0 b_1}{2} = 0, \\ \left(\frac{G'}{G}\right)^2: & \frac{k b_1}{4} + \frac{a_1 b_1}{2} = 0, \\ \left(\frac{G'}{G}\right)^0: & -a_0^2 + k\mu a_1 + \omega b_0 + \frac{3b_0^2}{4} + k_2 = 0, \\ \left(\frac{G'}{G}\right)^1: & k\lambda a_1 - 2a_0 a_1 + \omega b_1 + \frac{3b_0 b_1}{2} = 0, \\ \left(\frac{G'}{G}\right)^2: & k a_1 - a_1^2 + \frac{3b_1^2}{4} = 0. \end{aligned} \quad (3.1.12)$$

Solving this algebraic system gives

$$a_0 = \frac{1}{2}(\lambda a_1 - \omega), \quad a_1 = a_1, \quad b_0 = \lambda a_1 - \omega, \quad b_1 = 2a_1,$$

$$k_1 = \frac{1}{4}(\omega^2 - M a_1^2), \quad k_2 = \frac{1}{2}(\omega^2 - M a_1^2), \quad k = -2a_1 \quad (3.1.13)$$

Back-substitution of (3.1.13) into (3.1.10)–(3.1.11) yields

$$u(\xi) = \frac{1}{2}(\lambda a_1 - \omega) + a_1 \left(\frac{G'}{G}\right), \quad (3.1.14)$$

$$v(\xi) = \lambda a_1 - \omega + 2a_1 \left(\frac{G'}{G}\right), \quad (3.1.15)$$

$$\text{with } \xi = -2a_1(x + \omega t). \quad (3.1.16)$$

Setting $M = \lambda^2 - 4\mu$, the general solution of (2.5) takes one of three distinct closed forms according to whether M is positive, negative, or zero and (3.1.14)–(3.1.15) inherit this three-way split.

Case1 ($M > 0$): hyperbolic-function solution

$$u(\xi) = \frac{1}{2} \left[a_1 \sqrt{M} \left(\frac{A \cosh\left(\frac{1}{2}\sqrt{M}\xi\right) + B \sinh\left(\frac{1}{2}\sqrt{M}\xi\right)}{A \sinh\left(\frac{1}{2}\sqrt{M}\xi\right) + B \cosh\left(\frac{1}{2}\sqrt{M}\xi\right)} \right) - \omega \right], \quad (3.1.17)$$

$$v(\xi) = a_1 \sqrt{M} \left(\frac{A \cosh\left(\frac{1}{2}\sqrt{M}\xi\right) + B \sinh\left(\frac{1}{2}\sqrt{M}\xi\right)}{A \sinh\left(\frac{1}{2}\sqrt{M}\xi\right) + B \cosh\left(\frac{1}{2}\sqrt{M}\xi\right)} \right) - \omega, \quad (3.1.18)$$

Case2 ($M < 0$): trigonometric-function solution

$$u(\xi) = \frac{1}{2} \left[a_1 \sqrt{-M} \left(\frac{-A \sin\left(\frac{1}{2}\sqrt{-M}\xi\right) + B \cos\left(\frac{1}{2}\sqrt{-M}\xi\right)}{A \cos\left(\frac{1}{2}\sqrt{-M}\xi\right) + B \sin\left(\frac{1}{2}\sqrt{-M}\xi\right)} \right) - \omega \right], \quad (3.1.19)$$

$$v(\xi) = a_1 \sqrt{-M} \left(\frac{-A \sin\left(\frac{1}{2}\sqrt{-M}\xi\right) + B \cos\left(\frac{1}{2}\sqrt{-M}\xi\right)}{A \cos\left(\frac{1}{2}\sqrt{-M}\xi\right) + B \sin\left(\frac{1}{2}\sqrt{-M}\xi\right)} \right) - \omega, \quad (3.1.20)$$

Case3 ($M = 0$): rational solution

$$u(\xi) = a_1 \left(\frac{B}{B\xi + A} \right) - \frac{\omega}{2}, \quad (3.1.21)$$

$$v(\xi) = 2a_1 \left(\frac{B}{B\xi + A} \right) - \omega, \quad (3.1.22)$$

Specializing to $B = 0, A \neq 0, \lambda > 0, \mu = 0$ reduces (3.1.17)–(3.1.18) to

$$u(\xi) = \frac{1}{2} \left[a_1 \lambda \coth\left(\frac{\lambda}{2}\xi\right) - \omega \right], \quad (3.1.23)$$

$$v(\xi) = a_1 \lambda \coth\left(\frac{\lambda}{2}\xi\right) - \omega, \quad (3.1.24)$$

Alternatively, taking $B \neq 0$ with $A^2 > B^2, \lambda > 0, \mu = 0$ gives

$$u(\xi) = \frac{1}{2} \left[a_1 \lambda \tanh\left(\frac{\lambda}{2}\xi + \xi_0\right) - \omega \right], \quad (3.1.25)$$

$$v(\xi) = a_1 \lambda \tanh\left(\frac{\lambda}{2}\xi + \xi_0\right) - \omega, \quad (3.1.26)$$

with $\xi_0 = \tanh^{-1}(A/B)$. Equations (3.1.23)–(3.1.26) describe single-humped, localized wave profiles that decay away from a moving front in the solitary-wave regime of the MDWW system (3.1.1)–(3.1.2).

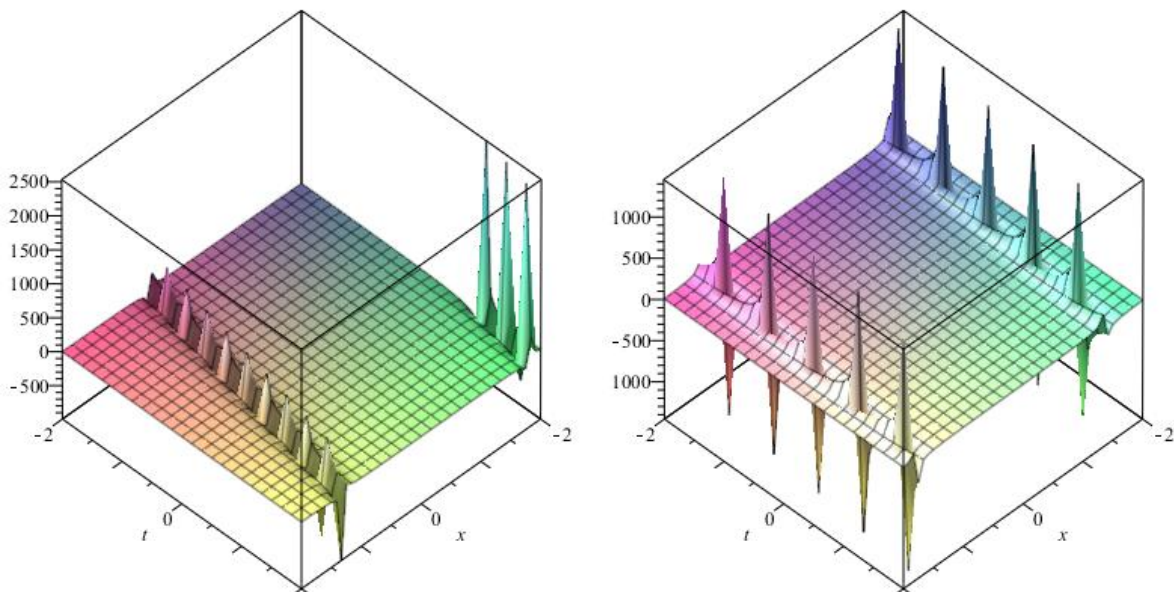


Figure 1. solution profiles (3.1.19), left, and (3.1.25), right, plotted with $a_0 = 2, \lambda = -3, \mu = -1, A = 3, B = 4, c = 5$.

Variant Boussinesq Equation [16]:

The variant Boussinesq system is a widely used framework for describing long nonlinear waves in dispersive media, combining nonlinear and dispersive effects within a single model. It allows the shape, stability, and interaction of such waves in fluid systems to be examined in more depth than linear wave theory permits, while remaining tractable for both analytical and numerical treatment.

$$u_t + v_x + uu_x = 0, \quad (3.2.1)$$

$$v_t + (uv)_x + u_{xxx} = 0, \quad (3.2.2)$$

Setting $\xi = x - ct$ (3.2.3) gives

$$-cu' + v' + uu' = 0, \quad (3.2.4)$$

$$-cv' + uv' + u'v + u''' = 0, \quad (3.2.5)$$

Balancing suggests $n = 1, m = 2$, so that

$$u(\xi) = a_0 + a_1 \left(\frac{G'}{G}\right), \quad a_1 \neq 0, \quad (32.6)$$

$$v(\xi) = b_0 + b_1 \left(\frac{G'}{G}\right) + b_2 \left(\frac{G'}{G}\right)^2, \quad a_2 \neq 0, \quad (32.7)$$

Substituting and matching powers of (G'/G) leads to the algebraic system

$$\begin{aligned} \left(\frac{G'}{G}\right)^0 &: C a_1 \mu - a_0 a_1 \mu - b_1 \mu = 0, \\ \left(\frac{G'}{G}\right)^1 &: C a_1 \lambda - a_0 a_1 \lambda - a_1^2 \mu - b_1 \lambda - 2 b_2 \mu = 0, \\ \left(\frac{G'}{G}\right)^2 &: -a_1^2 \lambda + C a_1 - a_0 a_1 - 2 b_2 \mu - b_1 = 0, \\ \left(\frac{G'}{G}\right)^3 &: -a_1^2 - 2 b_2 = 0, \\ \left(\frac{G'}{G}\right)^0 &: -a_1 \lambda^2 \mu + C b_1 \mu - a_0 b_1 \mu - a_1 b_0 \mu - 2 a_1 \mu^2 = 0, \\ \left(\frac{G'}{G}\right)^1 &: -a_1 \lambda^3 + C b_1 \lambda + 2 C b_2 \mu - a_0 b_1 \lambda - 2 a_0 b_2 \mu - a_1 b_0 \lambda - 2 a_1 b_1 \mu - 8 a_1 \lambda \mu = 0, \\ \left(\frac{G'}{G}\right)^2 &: 2 C b_2 \lambda - 2 a_0 b_2 \lambda - 2 a_1 b_1 \lambda - 3 a_1 b_2 \mu - 7 a_1 \lambda^2 + C b_1 - a_0 b_1 - a_1 b_0 - 8 a_1 \mu = 0, \\ \left(\frac{G'}{G}\right)^3 &: -3 a_1 b_2 \lambda + 2 C b_2 - 2 a_0 b_2 - 2 a_1 b_1 - 12 a_1 \lambda = 0, \\ \left(\frac{G'}{G}\right)^4 &: -3 a_1 b_2 - 6 a_1 = 0, \end{aligned} \quad (32.8)$$

Three admissible branches follow:

$$C = C, \quad a_0 = a_0, \quad a_1 = 0, \quad b_0 = b_0, \quad b_1 = 0, \quad b_2 = 0, \quad (32.9) \quad C = a_0 - \lambda, \quad a_0 = a_0, \quad a_1 = 2, \quad b_0 = -2\mu, \quad b_1 = -2\lambda, \quad b_2 = -2, \quad (32.10)$$

$$C = a_0 + \lambda, \quad a_0 = a_0, \quad a_1 = -2, \quad b_0 = -2\mu, \quad b_1 = -2\lambda, \quad b_2 = -2, \quad (32.11)$$

Taking the non-trivial branch (3.2.10) gives

$$u(\xi) = a_0 + 2 \left(\frac{G'}{G}\right), \quad (32.12)$$

$$v(\xi) = -2\mu - 2\lambda \left(\frac{G'}{G}\right) - 2 \left(\frac{G'}{G}\right)^2, \quad (32.13)$$

$$\text{with } \xi = x - (a_0 + \lambda)t, \quad (32.14)$$

Using the general solution of (2.5) according to the sign of M , the following solution families are obtained.

If $M > 0$:

$$u(\xi) = a_0 + 2 \left[\frac{1}{2} \sqrt{M} \left(\frac{A \cosh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \sinh\left(\frac{1}{2} \sqrt{M} \xi\right)}{A \sinh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \cosh\left(\frac{1}{2} \sqrt{M} \xi\right)} \right) - \frac{\lambda}{2} \right], \quad (32.15)$$

$$u(\xi) = a_0 + \sqrt{M} \left(\frac{A \cosh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \sinh\left(\frac{1}{2} \sqrt{M} \xi\right)}{A \sinh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \cosh\left(\frac{1}{2} \sqrt{M} \xi\right)} \right) - \lambda, \quad (32.16)$$

$$v(\xi) = -2\mu - 2\lambda \left[\frac{1}{2} \sqrt{M} \left(\frac{A \cosh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \sinh\left(\frac{1}{2} \sqrt{M} \xi\right)}{A \sinh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \cosh\left(\frac{1}{2} \sqrt{M} \xi\right)} \right) - \frac{\lambda}{2} \right] - 2 \left[\frac{1}{2} \sqrt{M} \left(\frac{A \cosh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \sinh\left(\frac{1}{2} \sqrt{M} \xi\right)}{A \sinh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \cosh\left(\frac{1}{2} \sqrt{M} \xi\right)} \right) - \frac{\lambda}{2} \right]^2, \quad (32.17)$$

$$v(\xi) = -2\mu + \frac{\lambda^2}{2} - \frac{M}{2} \left(\frac{A \cosh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \sinh\left(\frac{1}{2} \sqrt{M} \xi\right)}{A \sinh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \cosh\left(\frac{1}{2} \sqrt{M} \xi\right)} \right)^2, \quad (32.18)$$

If $M < 0$:

$$\begin{aligned} u(\xi) &= a_0 + 2 \left[\frac{1}{2} \sqrt{-M} \left(\frac{-A \cos\left(\frac{1}{2} \sqrt{-M} \xi\right) + B \sin\left(\frac{1}{2} \sqrt{-M} \xi\right)}{A \sin\left(\frac{1}{2} \sqrt{-M} \xi\right) + B \cos\left(\frac{1}{2} \sqrt{-M} \xi\right)} \right) - \frac{\lambda}{2} \right], \\ u(\xi) &= a_0 + \sqrt{-M} \left(\frac{-A \cos\left(\frac{1}{2} \sqrt{-M} \xi\right) + B \sin\left(\frac{1}{2} \sqrt{-M} \xi\right)}{A \sin\left(\frac{1}{2} \sqrt{-M} \xi\right) + B \cos\left(\frac{1}{2} \sqrt{-M} \xi\right)} \right) + \lambda, \end{aligned} \quad (32.19)$$

$$v(\xi) = -2\mu - 2\lambda \left[\frac{1}{2} \sqrt{-M} \left(\frac{-A \cos\left(\frac{1}{2} \sqrt{-M} \xi\right) + B \sin\left(\frac{1}{2} \sqrt{-M} \xi\right)}{A \sin\left(\frac{1}{2} \sqrt{-M} \xi\right) + B \cos\left(\frac{1}{2} \sqrt{-M} \xi\right)} \right) - \frac{\lambda}{2} \right] - 2 \left[\frac{1}{2} \sqrt{-M} \left(\frac{-A \cos\left(\frac{1}{2} \sqrt{-M} \xi\right) + B \sin\left(\frac{1}{2} \sqrt{-M} \xi\right)}{A \sin\left(\frac{1}{2} \sqrt{-M} \xi\right) + B \cos\left(\frac{1}{2} \sqrt{-M} \xi\right)} \right) - \frac{\lambda}{2} \right]^2,$$

$$v(\xi) = -2\mu + \frac{\lambda^2}{2} - \frac{(-M)}{2} \left(\frac{-A \cos\left(\frac{1}{2} \sqrt{-M} \xi\right) + B \sin\left(\frac{1}{2} \sqrt{-M} \xi\right)}{A \sin\left(\frac{1}{2} \sqrt{-M} \xi\right) + B \cos\left(\frac{1}{2} \sqrt{-M} \xi\right)} \right)^2, \quad (32.20)$$

If $M = 0$:

$$u(\xi) = a_0 + 2 \left[\frac{B}{B\xi+A} - \frac{\lambda}{2} \right], \quad (.32.21)$$

$$u(\xi) = a_0 + \frac{2B}{B\xi+A} - \lambda, \quad (.32.22)$$

$$v(\xi) = -2\mu - 2\lambda \left[\frac{B}{B\xi+A} - \frac{\lambda}{2} \right] - 2 \left[\frac{B}{B\xi+A} - \frac{\lambda}{2} \right]^2, \quad (.32.23)$$

$$v(\xi) = -2\mu + \frac{\lambda^2}{2} - 2 \left(\frac{B}{B\xi+A} \right)^2, \quad (.32.24)$$

If $M > 0$:

$$u(\xi) = a_0 - 2 \left[\frac{1}{2} \sqrt{M} \left(\frac{A \cosh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \sinh\left(\frac{1}{2} \sqrt{M} \xi\right)}{A \sinh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \cosh\left(\frac{1}{2} \sqrt{M} \xi\right)} \right) - \frac{\lambda}{2} \right], \quad (.32.25)$$

$$v(\xi) = -2\mu - 2\lambda \left[\frac{1}{2} \sqrt{M} \left(\frac{A \cosh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \sinh\left(\frac{1}{2} \sqrt{M} \xi\right)}{A \sinh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \cosh\left(\frac{1}{2} \sqrt{M} \xi\right)} \right) - \frac{\lambda}{2} \right] - 2 \left[\frac{1}{2} \sqrt{M} \left(\frac{A \cosh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \sinh\left(\frac{1}{2} \sqrt{M} \xi\right)}{A \sinh\left(\frac{1}{2} \sqrt{M} \xi\right) + B \cosh\left(\frac{1}{2} \sqrt{M} \xi\right)} \right) - \frac{\lambda}{2} \right]^2 \quad (.32.26)$$

If $M = 0$:

$$u(\xi) = a_0 - 2 \left[\frac{B}{B\xi+A} - \frac{\lambda}{2} \right], \quad (.32.27)$$

$$u(\xi) = a_0 - \frac{2B}{B\xi+A} + \lambda, \quad (.32.28)$$

$$v(\xi) = -2\mu - 2\lambda \left[\frac{B}{B\xi+A} - \frac{\lambda}{2} \right] - 2 \left[\frac{B}{B\xi+A} - \frac{\lambda}{2} \right]^2, \quad (.32.29)$$

$$v(\xi) = -2\mu + \frac{\lambda^2}{2} - 2 \left(\frac{B}{B\xi+A} \right)^2, \quad (.32.30)$$

The mirror branch (3.2.11) reproduces the same three cases with $a_1 = -2$, giving the sign-reversed counterparts of (3.2.25)–(3.2.30) in the original derivation

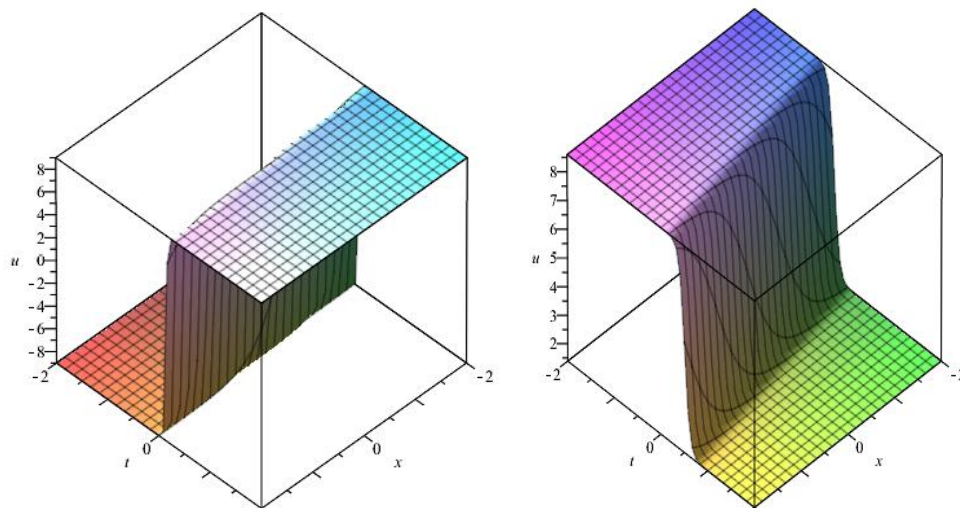


Figure 3.2a solution profiles (3.2.17), left, and (3.2.16), right, plotted with $a_0 = 2$, $\lambda = -3$, $\mu = -1$, $A = 3$, $B = 4$, $c = 5$.

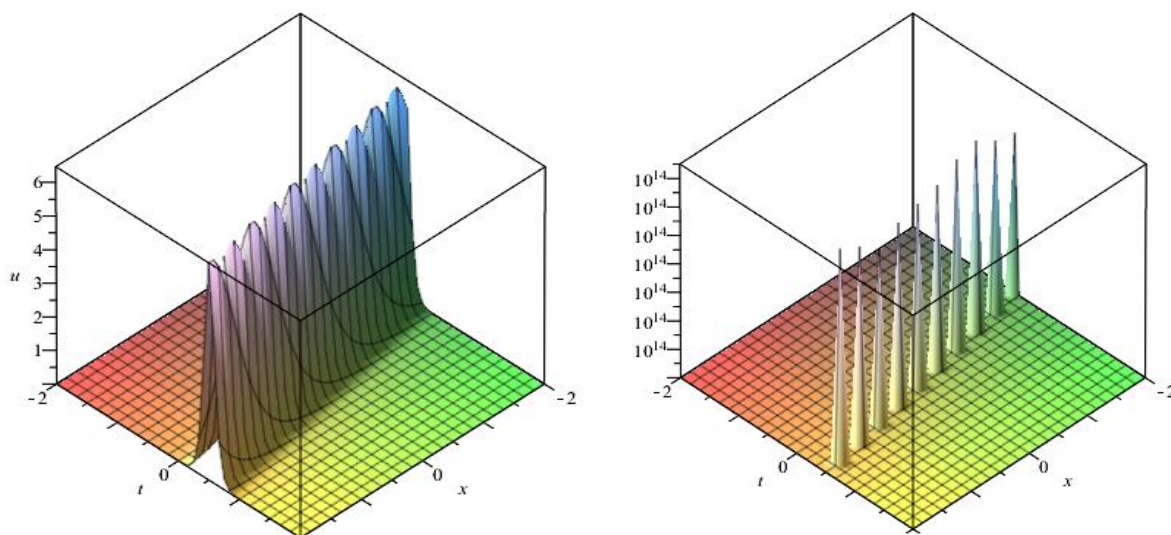


Figure 2. Solution profiles (3.2.19), left, and (3.2.22), right, plotted with $a_0 = 2$, $\lambda = -3$, $\mu = -1$, $A = 3$, $B = 4$, $c = 5$.

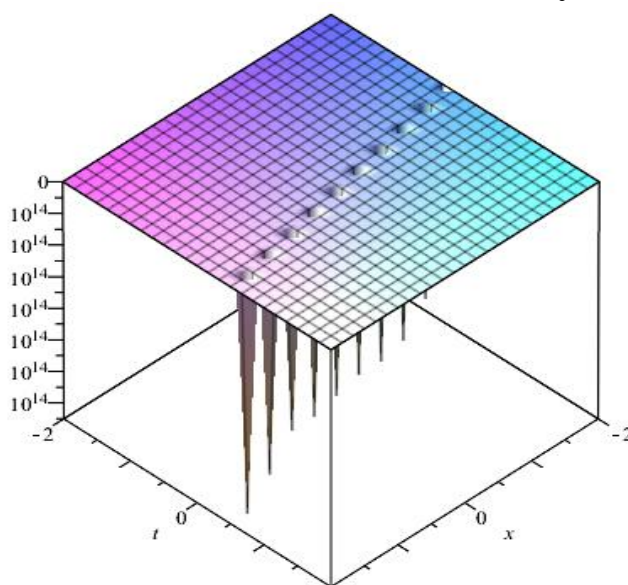


Figure 3. The (3.2.30) solution profile, plotted with $a_0 = 2$, $\lambda = -3$, $\mu = -1$, $A = 3$, $B = 4$, $c = 5$.

Conclusion

Two nonlinear model systems the MDWW equations and the variant Boussinesq equation were treated here with the (G'/G) -expansion procedure, yielding several previously unlisted closed-form traveling-wave families for each. Working through both cases shows that the algebra involved stays manageable even as the number of solution branches grows, which speaks to the practicality of the method rather than just its formal validity. A natural continuation would be to test the same procedure against other nonlinear PDEs encountered in mathematical physics and applied analysis; that extension is left open for later work.

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